Introduction to Astrophysics

TEST #1

Feb 25, 2025

1. Why is the virial theorem so important in astrophysics? What quantities does it connect?
2. When applying the virial theorem to stars, we assumed that they are in hydrostatic equilibrium. Justify that assumption from the observational viewpoint.
3. Consider a spherically symmetrical cloud that collapses into a protostar. Show that the net amount of radiated energy is $3GM^2/10R$.
4. What is the difference between an isothermal and homologous collapse of the nebula? Which (if any) better represents reality?
5. Show that, for an adiabatic process (i.e., $pV^\gamma = \text{const.}$), the relationship between temperature and density is $T \propto \rho^{\gamma-1}$.
6. Calculate the minimum mass of the cloud made of equal parts hydrogen and helium, temperature 150 K and density 10^6 particles per cm^3, to collapse into a protostar. Express your answer in solar masses.
7. Name at least 5 physical circumstances that we neglected in our analytical description of protostar formation.
8. What does the Kelvin-Helmholtz timescale correspond to? Derive the expression for t_{KH}. What is the value of t_{KH} for a Sun-like star? Could it sustain the Sun throughout its lifetime?
9. Explain in words how do we obtain the Lane-Emden equation. What does it represent?
10. What is the role of the polytropic index n in Lane-Emden's equation? Show how the equation is solved for $n = 0$.

Relevant physical constants:

Boltzmann constant	$k = 1.38 \times 10^{-23}\,\text{J/K}$
Gravitational constant	$G = 6.67 \times 10^{-11}\,\text{Nm}^2/\text{s}^2$
Hydrogen mass	$m_H = 1.67 \times 10^{-27}\,\text{kg}$
Mass of the Sun	$M_\odot = 2 \times 10^{30}\,\text{kg}$
Luminosity of the Sun	$L_\odot = 6 \times 10^{24}\,\text{W}$