Autor: Danny Jensen

Title: A Computational Universal Curvature Fitting Algorithm

Research Summary:

This paper presented a preliminary version of a new algorithm for fitting a curvature model to the universe called CURVFAM. This algorithm implements a top-down approach, assuming known properties of the universe and applying a function to those values to determine observables, specifically, distances and angles between objects. This code assumes a curvature metric K, which is a separable function of space and time. In reality, K is best modeled as a series of exponentials with variable coefficients. However, this paper assumes K to be a series of polynomials. This exaggerates the results of applying curvature to better demonstrate the capabilities of CURVFAM. The paper first presents a simplified one-dimensional application of CURVFAM to demonstrate how changing curvature affects observables. In this case, K simplifies to a function of one special dimension and no time dependence. Flat space is assumed to be a uniform distribution of points on a straight line. Once curvature is applied, the distribution of distances from a zero-reference point becomes largely right-skewed, and the distances substantially increase. Next, the paper extends this to the higher-dimensional case, where K becomes a series of three equations, each describing one special dimension – r, theta, or phi. Similar to the one-dimensional case, the distribution of distances from a zero-reference point is symmetrical in flat space, but becomes right skewed in curved space, and the distances increase. Also, the distribution of angles between adjacent points is a narrow, symmetric distribution in flat space, but becomes right skewed in curved space, and the angles increase. In future implementations of CURVFAM, a fitter will be added to determine the curvature of the universe from observational data, and General Relativity will be considered.

Recommendation: Accept with minor revision

The methodology seems to be executed correctly and the results are reasonable. The main issue with this paper is clarification. There are many instances where things needed to be explained in more detail or reasoning needed to be elaborated.

Concerns:

Major points:

- In Figure 1, the author should mention if a particular type of curvature (open or closed space) is used to get the desired effect. Also, the curved space representation (right) is unclear. It is unclear what D_1 and D_2 are in this context (the true distances or the distances measured as a result of curvature). D_1 is drawn from the reference to the true location, but D_2 is drawn from the reference to the observed location. The figure needs to be changed to better clarify this. Also, to better match the left (observables measured from a reference point), D_1 should be on the left of the angle and D_2 should be on the right.

- The author needs to mention what curvature function is used explicitly in the methodology. Much of the introduction relies on the explaining the different types of curvature, but it is unclear if K is assumed to be for an open or closed universe. In addition, the explicit equations for K that are used should be stated. The author mentions a series of polynomials, but at the very least, the degree should be stated.
- It should be explained in the introduction that K is best modeled as a series of exponentials with variable coefficients and why this is the case. Also, the author should explain in the methodology why using a series of polynomials as an example function is a valid step to take when this is not the case in reality.
- Equation 3 is unclear. Each equation for K demonstrates dependence on one spatial dimension but is still a function of Y(r). It might be more correct to say Y(r), Y(θ), and Y(φ) respectively, but it is not clear if this is the case. If this is not the case, then it is unclear how the dependence on one spatial dimension plays a role when K is still a function of a general vector r in each equation. And again, the explicit equation for K that was used in the methodology should be stated here.
- The author mentions that, in general, the distribution of angles "will be observed in a spreading of the distribution and an increase in the mean angle." It should be clarified if this will always be the case, and if not, if there is an example where this would not be the case. I.e., is this a result of the explicit curvature function chosen for the methodology, or is this case for all open/closed curvature, etc.?
- The paper might benefit from explicitly stating the mean angles and distances that are presented in the histograms, either in the text or in a table, to give a better quantitative report of the results.

Minor points:

Introduction:

- Include a short introductory paragraph explaining why it is important to measure the curvature of the universe and what the implications of this are for other areas of astronomy. Paragraph 5 can be moved and combined with this introduction.
- In paragraph 2, it is unclear if the FLRW metric is different from the "equations that govern the shape of the universe." Reword the sentence the clarify this.
- In paragraph 3, it is unclear if the curvature parameter K distinct from the "equations that govern the shape of the universe." If it is, the author should explain why determining K be more appealing than solving these equations.
- At the end of paragraph 4, the author mentions "tension in the ultimate determination of the global curvature of the universe between the various methods" but does not explain what this tension is or why it exists.

Theory:

- The first paragraph seems unnecessary. There is no reason to explain what a bottom-up approach entails if it is never referred to again.
- In paragraph 2 (before Equation 1), change to “a function of both time (T(t)) and space (Y(r))” to better clarify Equation 1.

1-D Model:

- It should be explicitly stated that that reference point is set at zero/the origin.

Comprehensive Model:

- The distribution of points is homogeneous and isotropic. This choice should be explained because the distribution of galaxies, for example, is not. And, if CURVFAM is meant to be used with supernovae in various distant galaxies, a non-uniform distribution might be expected.
- The author mentions that r is essentially the same as time because it represents the time it takes light to travel to the reference point. However, it is unclear if this is different from the T(t) dependence in Equation 3.
- Figure 4 is best placed after paragraph 3.
- The last sentence of paragraph 4 can be cut since this is mentioned in the figure caption and makes the transition to the next paragraph confusing.
- The last paragraph can be cut and replaced with a brief summary of the results and an overview of the capabilities of CURVFAM that were demonstrated in the paper.

Evaluation of Criteria:

Is the length appropriate?

There are some clarifications or explanations that need to be added (according to the comments above). Revisions may increase the length of the paper, but it should not be much longer than its current length.

Are the title and abstract sufficiently informative?

The abstract could be a bit clearer that this paper is presenting a preliminary/prototype version of CURVFAM, and that the results presented are not from the finalized code.

Is the contribution to science significant?

Yes, CURVFAM presents a computationally easier way to model the curvature of the universe from observations and has implications for cosmology and measuring important cosmological parameters such as the Hubble constant, the matter density parameter, and the dark energy density parameter.

Is level of English adequate?

There are a lot of instances where clarification or reorganization is needed in order to follow the narrative presented in the paper.

Is the literature properly cited?

Yes, the work is properly grounded in literature.

Are the results clearly and accurately presented?

Results such as the mean angle values and mean distance values presented in the histograms could also be presented in a table or stated in the text to better articulate how these values change when curvature is applied.

Is the topic appropriate for the journal?

Yes, this is appropriate for the journal.

Data Management Plan:

The results presented in the paper are not exactly reproducible. The graphs and results rely heavily on the curvature function used, but the exact function is never explicitly stated. This makes it difficult for readers to corroborate results with those presented in this paper. However, future implementations and versions of CURVFAM will be made available online and will be made user friendly. Even further implementations will be presented in a web-based format. This will make the algorithm easily accessible and usable to many scientists.

Peer review report

Title: A Computational Universal Curvature Fitting Algorithm

Author: Danny Jensen

Summary

The author introduces CURVFAM (CURVature Fitting AlgorthM) to model a universe under a prescribed curvature metric. In a curved space, distance and apparent angle between two objects will change according to the curvature. A bottom-down approach is used for the code, in which different curvature metric are applied to find which one matches with the observables. Firstly, the author uses a simple one-dimensional model to see the impact of curvature on the observable geometry. As a result, the distribution of distances from reference point in the curved space is significantly less uniform from that in place space. With the foundation on 1-D space, the author develops the code into 3-D model. In this case, the curvature metric is a series of three equations, each describing the curvature of φ, θ,and r in the spherical coordinate. The author proves that in curved space, the distribution of angles between adjacent points spreads out more compared to the case with flat space and the mean angle also increases in the curved space. Moreover, the distribution of distances from the reference point is influenced by the curvature, losing the symmetrical distribution as in the case with flat space. Lastly, the author emphasizes the future implementation of CURVFAM. Specifically, the observational data will be used in the code, and then fitter is utilized to find the best curvature function. General relativity effect will also be included in the implementation. New modeling methods and fitting algorithms will be continually tested to improve the code's efficiency, and a more user friendly GUI will be created for easier access.

Recommendation: accepted with minor revisions.

Justification

a. **Major points**

- 1-D Model:
  - *"… by applying a prescribed curvature function to their location."*: the author should specify what curvature function he uses to produce Figure 2.

- Comprehensive model:
  - *"The prescribed curvature metric is applied…"* and "*It is important to note that the exact results depend on the curvature functions used*": As the author point out, each curvature function will result in a different result. Thus, it is important to specify what curvature function he uses to test the comprehensive model in this section.
  - *"… and average angle value from the same pairs in curved space"*: the author should explain how he calculates the angle in curved space.

b. Minor points

- Introduction:
  - *"The two main methods used for the determination … the cosmic microwave background (CMB)"*: need citation.

- Theory
  - Figure 1: the author should explain what D_1, D_2, φ_1, and φ_2 are in the right figure. Are D_1 and D_2 the same as in the left figure?

- 1-D Model
  - *"… uniform spacing and density to a less uniform distribution…"*: the author should give more explanation for the distribution in the curved space. For example, why is there a higher density in the smaller distance? And why is there a uniform distribution starting from the distance 100?
  - Figure 3: the author lists the unit of the distance as "units". What is the length of this "units" in relation to Figure 2?

- Comprehensive model
  - Figure 4: the author put the title of the right figure as "Position in Curved Space". Is it positively curved or negatively curved?
  - Figure 5 and 6: the author should describe and give more explanation of the distributions for the case of curved space. For example, what causes the spreading out of the distribution of angles? Why does the mean angle increase, and does this increasing depend on the type of curvature (positive and negative)? Or in Figure 6, why are there a large density in the lower distance range, and how are there two local maximums at about 10 and 24 distance units.

- Future Implementations
  - *"With the large amount of data…"*: the author should specify what type of data he mentions here.

Evaluation

The length of the paper is appropriate and the topic is appropriate for the journal. The title and the abstract convey the content of the study. The contribution to science is significant. The English in the paper is easy to follow, however, some sentences have repetitive words that need to be rephrased better ("Attempt range …attempting to solve", or "The two main methods used for … parameter K use…"). One citation is missing, but overall the paper has good citation. More explanations are needed for the results rather than just figures. Regarding the Data Management plan, the author does not mention where or if he will publish his code for public access.

All my comments are constructive reviews and sometimes they can come from personal preference or personal experience. Please understand if I misunderstand the author in some points. Also, please send my congratulation on the author's work and I enjoy reading the paper.

Author: Danny Jensen

Title: A Computational Universal Curvature Fitting Algorithm

Summary:

This paper is an intro paper to a new program called CURVFAM (CURVature Fitting AlgorithM). This code is used to make universe simulations given a prescribed curvature metric. The paper starts with an introduction explaining different types of curvature, the previous attempts to determine the shape of the universe and the growing use of computational methods. Next the theory is described including a description of the two main approaches to determining universal properties. The author adopts the top-down approach to determine curvature functions. This section describes how observables (distance and angle) change in curved space. The following section introduces a 1-D model before moving to the more complex 3-D model. It is shown how distance from a reference point will change when curvature is applied, and Figure 3 graphically demonstrates how observed distances will change. Next the full three-dimensional model is explained along with an example to display the effects of applied curvature. Now, instead of just perceived distance, the angle between two adjacent points change as well. Lastly, the paper discusses the future plans for the program CURVFAM.

Rating: Accept with major revision

Minor Revisions:

- Two examples are used, one for the 1-D case and another for the 3-D case. It is explained that the functions applied are polynomials, however I feel that the actual curvature function should be displayed, even if just in the figure captions.

- The image in Figure 1 (right) could be better explained. The figure is confusing and how the different distances and angles change with applied curvature should be explained better.
- In the Comprehensive Model section, on page 5, the concept of the *r* component being representative of time seems random. I do not see how this relates to the task of computing the new observables in curved space. This should be better explained

Major Revisions:

- The author does not give a detailed explanation of how observables are computed in CURVFAM. For example, the reader does not know what CURVFAM does under the hood to go between the two plots displayed in Figure 4. The reason for this paper is to introduce a new program to be used by other astronomers, which means there must be emphasis put into how the program works. A section detailing the algorithm and equations used should be added since there is no mention of how CURVFAM computes the new location of a point in curved space.
- There is no way for a reader to download or work with this code. The code should be uploaded to GitHub or some other online source for a reader to use. One of the main goals set by the author is to have CURVFAM publicly released to be used by other astronomers, but I feel that this goal is not met. It is stated that future implementations will include a fitter and a GUI. While those are still in the works, the initial code used for computing observables should be made publicly available. Otherwise, this paper is published as an introduction to CURVFAM while giving the reader no way to work with the program.

Evaluation of Criteria:

Is the length appropriate?

- The paper seems short, especially since this paper is introducing a new program. The theory section seems short and there should be more emphasis on how CURVFAM works under the hood. An additional section describing the equations and algorithm utilized by CURVFAM would benefit the reader.

Are the title and abstract sufficiently informative?

- Yes, my only comment is that the name of the program (CURVFAM) could be mentioned in the title so readers will recognize this as the introduction paper to the code.

Is the contribution to science significant?

- Yes, CURVFAM offers a great benefit to the field of cosmology. Future implementations described in the paper will be beneficial to fitting universal data. I think that CURVFAM has pedagogical significance as well, but this aspect is not discussed in the paper. This program does a great job of visually demonstrating how curvature impacts observables and I think this could be a great teaching tool. I think mentioning this would benefit the merit of CURVFAM as a science contribution and a contribution to science teaching.

Is the level of English adequate?

- Yes

Is the literature properly cited?

- Yes, the background theory is rooted in the literature. However, in the abstract, it is mentioned that there are other universal simulators. While the differences that CURVFAM offers is explained, the other simulators, and their citations, should be provided.

Are the results clearly and accurately presented?

- The examples provided by the author in 1-D and 3-D are well presented with plots. However, I feel that a result of this study is CURVFAM itself and as mentioned earlier, the program is not released to the program and thus not clearly presented.

Is the topic appropriate for this journal?

- Yes

Data Management Plan?

- There are no numerical results to be presented. However, as stated before, the code for CURVFAM needs to be accessible. The fitting software and GUI are future works. For now, CURVFAM exists as a code that must be directly modified. Therefore, GitHub would be a good location to make the code public

Additional Comments:

This works is extremely interesting. I would like to thank the author for all the hard work they have put into this study. I think that CURVFAM has amazing potential.

Danny Review

Summary:

The author describes their development of the CURVature Fitting AlgorithM (CURVFAM). This code models a generic and uniform Universe under the presence of a curvature function. The point of the model is to understand what kind of observables would be notable in the Universe depending on how it is curved. Currently, the code populates a model Universe with uniformly distributed points in a volume and then applies curvature in the form of a given function. From here, the author is able to produce plots describing how the distribution of angles between adjacent points and the distribution of distances from the reference point change from non-curved to curved space. The author suggests that further developments of this code will allow take general relativity into account, analyze real, observational data, and be formatted as a GUI or even web-interface. Overall, once further developed, CURVFAM will aid in determining the curvature of the Universe through comparison of these models to observational data.

Recommendation:

I would recommend that this paper be **accepted with minor modifications**.

Justification:

- *Major Points:* None.
- *Minor Points:*
    - Introduction: "…supernova surveys with solutions to the FLRW metric across various tested values for the density parameters of matter…and dark energy…"
        - Since the author introduced these ideas of density parameters, perhaps the paper would benefit from a definition and/or explanation of them.
        - Would be especially helpful because the concept comes up again later, toward the end of the Introduction section.
    - Theory: "…assuming that the curvature metric (*K*) can change as a function of both time and space."
        - The equation following this sentence is presented as a key point, but none of the variables other than *K* are explained!
        - It may also be helpful to explain the significance of this equation moving forward- it seems that the connection between the equation and the theory are evident to the author, but this is less of the case for the reader.
        - In other words, following the equation perhaps the author could explain that the variables in Eq (1) represent the observables in the Universe.
    - Figure 1:
        - This figure is somewhat confusing when explaining the concept of simple curved space.

    - Perhaps the left image should simply be copied over to the right one and then adjusted to account for curvature instead of changing the nature of the initial premise of the left figure.
    - Do D1 and D2 mean the same thing in the left vs. the right image?
    - Are the true positions of the objects seen as the dashed lines or the dots on those lines?
    - Maybe the use of different colors would have made this figure easier to understand?
  - <u>Equation (2):</u>
    - What does S_i represent in this equation?
    - It is also unclear whether Eq (2) is the distance formula along a curve.
    - Was this equation derived by the author or does it have a reference?
  - <u>Figure 3:</u>
    - What curvature function was used for the right plot?
  - <u>Comprehensive Model: "Equation 3 shows how the curvature metric is broken into three separate functions where K_i describes the curvature of space in the ith dimension."</u>
    - Introduction of K_i, but never any use of it.
    - Does Eq (3) refer to a 3-D model, or 4-D?
  - <u>Comprehensive Model: "…while the *r* component of the spherical coordinates is representative of the radial distance from the reference point, this is synonymous with time due to the finite speed of light."</u>
    - Possibly beneficial to elaborate on this topic.
    - Does this mean that the 3-D model is the same as the 4-D model?
    - Is it possible to have radial compression or expansion without time also being compressed or expanded?
  - <u>Figure 4:</u>
    - It may be helpful to decrease the point size for these figures so that instead solid masses which are not easily shape-distinguished, the reader can see distinct shapes.
  - <u>Conclusion:</u>
    - A mere suggestion- but perhaps the author should add a discussion of the possibility that space is not curved the same way everywhere. Additionally, it is possible that space is curved at random and would not produce a specific series of observables as this research seems to imply.
    - It may also be important for the author to note that CURVFAM models potential observables with the caveat that the observed objects are uniformly distributed throughout the corresponding non-curved space. Would the algorithm (in future developments) benefit from modeling random points in Cartesian space and then adding a curve?

Evaluation:

- *Is the paper length appropriate?*

    - At times, the paper would have benefitted from additional explanations and connections (as discussed in the *Justification* section). This being the case, perhaps a slightly longer paper would be an improvement.
- *Are the title and abstract sufficiently informative?*
    - Yes! The title and abstract both accurately present and address the purpose of the research. The abstract appropriately summarizes the current state of the proposed software and its future implications.
- *Is the contribution to science significant?*
    - Yes. The research is in response to a currently debated topic in science, and will have a very impactful role on the future of computational universe modeling software.
- *Is the level of English adequate?*
    - Yes. Very easy to read and understand. Complex topics are explained both accurately and simply for the non-expert reader.
- *Is the work properly grounded in literature?*
    - Yes. Appropriate credit has been given to those who have pioneered this field, and there is significant acknowledgement of others who have addressed the same problem in different ways.
- *Are the results clearly and accurately presented?*
    - Yes. The current state of CURVFAM is accurately presented through the use of plots from an example curvature of the Universe.
- *Is the topic appropriate for the Journal?*
    - Yes. The author's research is a clear fit for this Journal being on the topic of Universe topology.
- *Is the data management plan good?*
    - Yes. The author clearly states their intent on making CURVFAM available to the public and also suggests the use of certain data for future use in the program.

Final Comments:

This paper was a fascinating read! It has hopes of beginning a new stage to figuring out the topology of the Universe through the use of modeling and analysis methodologies. I hope these comments are accepted as constructive criticisms and the author finds them helpful. My best wishes in their success!

Title: A Computational Universal Curvature Fitting Algorithm

Author: Danny Jensen

Summary: The author introduces the study by providing background on the types of curvature (flat, open, and closed) for an object. Many research groups have attempted to determine the shape of the universe over time by looking at supernovae and the cosmic microwave background, but there remains disagreement regarding the universal parameters. Computers also play a role in this debate by allowing astronomers to create models and simulations of a universe. It is important to design a simulation with a known curvature to save time and resources. The author's goal was to determine the global curvature of the universe. He applies a top-down approach for the curvature function by assuming the curvature metric depends on time and space. Next, he creates a 1D model with the observable of the distance from the reference point along the direction of space. He extends the model to a 3D case by starting with a flat, homogeneous, and isotropic distribution then applying a curvature metric. He considers the observables of the distance to the object and the angle formed between two objects through the reference point. In future studies, he hopes to finalize a fitting component for the models he created in this study.

Recommendation: Accept after major revisions.

Justification of Recommendation:

Major points:

- Throughout the paper, the author mentions that "curvature is applied." The methodology of the paper would be made clearer by stating what exactly is changed in the code when a curvature is applied.
- The paper would benefit from a discussion section that clearly reviews the implications/ importance of the results from this study. The directions for future research seem more obvious to a reader than the results from this study.
- The author does not provide directions for accessing the source code. It should be stated whether or not this code is available to the public. If the source code is available for public use, then he should include where/ how to access it.

Minor points:

- The third paragraph of the introduction starts off by mentioning the curvature parameter K. It would help if there was some explanation on what K actually refers to in terms of the shape or future of the universe.
- The first sentence of the caption for Figure 2 is a little confusing to read. Making it into a complete sentence would help with this.
- The title is somewhat misleading because the study does not actually create a fitting algorithm.

<u>Evaluation:</u>

1. Is the length appropriate?
    a. Yes – the length is appropriate, but some parts could use a more in-depth explanation (see justifications above).
2. Are the title and abstract sufficiently informative?
    a. As mentioned in the minor points, the title is misleading. The abstract provides a concise but informative overview of the study.
3. Is the contribution to science significant?
    a. Yes – the project addresses a question that faces debate in astronomy. It will be useful for other astronomers to see the foundation that this study presents in order to build off of it in future ones.
4. Is the level of English adequate?
    a. Yes – the level of English is adequate.
5. Is the literature properly cited?
    a. Yes – the author sufficiently cites necessary resources.
6. Are the results clearly and accurately presently?
    a. The results are presented accurately. However, they could be discussed more clearly (see justification section).
7. Is the topic appropriate for this journal?
    a. Yes – the topic is appropriate for this journal.
8. Data management plan?

a. The author explains directions for future work using CURVFAM, but he should include the location of the source code if it is available for others to use.

Additional Comments: Kudos to the author for engaging in a topic of strong debate in the field. He takes successful first steps in tackling the curvature question and provides a clear foundation for himself (and/ or others) to continue with his exciting work.

Author: Danny Jensen

Title: A Computational Universal Curvature Fitting Algorithm

Summary:

The author introduces the method by which the study will be carried out by acknowledging that computational modeling has become such a good resource due to advancing technology. Through computational modeling, the author introduces the idea of different models of curvature in which the universe can exist: flat, open, and closed. The curvature can be measured by measuring the angles of a hypothetical triangle in each model. In each different universe, the sum of the angles are all different. From these models, parameters such as Hubble's constant, dark matter, and regular matter density can be further analyzed alongside the shape of the whole universe. The author explains that the method incorporated into the study will include a number of particles that will model the universe and utilize a top-down approach in which the properties of the universe are known, to see what values the observables will take on. The author utilizes the model to show the differences between particles in one dimension versus curved space. In the comparisons, the code depicts the difference in the distance to an object and the angle that forms between two points through a reference point. These are the observables that the author meant to showcase.

Recommendation: Accept with minor modifications.

Justification

Major Point

- While it is a difficult task, the code successfully depicts the difference within the arrangement of the particles in different curvature models of the universe, but has not faced a more realistic scenario. The author does mention this will occur pending a future update on the code.

Minor Points:

- While the author mentions future iterations of the code will be rolled out, there is no mention of a timestamp of when users can expect these updates to become accessible. Perhaps a few sentences can clear this confusion up.

Evaluation:

1. Length
    a. The length of the paper is appropriate and does not hinder any sort of understanding.
2. Title and Abstract
    a. The title is sufficiently informative as it gives insight into what the study is about. The abstract, while informative, does not mention the different results the code has determined under the scenarios the author poses throughout the paper.
3. Science Contribution
    a. Yes, the contribution is significant as it would help develop the current understanding of the curvature of the universe. Upon future additional implementation to the code, real data could potentially be analyzed.
4. English level
    a. Yes, the level of English is adequate.
5. Literature Citation
    a. Yes, the literature is properly cited throughout the paper.
6. Results
    a. The results
7. Is the topic appropriate?
    a. Yes, the topic is appropriate.
8. Data Management Plan
    a. The author makes mention of the code becoming available for public use as additional modifications are made to the code.

Additional Comments: This was definitely one of the most interesting topics that has been tackled and to make significant progress within a short period of time is very impressive.

Summary

There have been many attempts to figure out the curvature of the universe, like solving the FLRW equations and creating statistical models. This author presents a new method to determine the curvature of the universe by using a curvature fitting algorithm based on the top down approach that the properties of the universe are known and functions can be applied to explain its geometry. The main idea of this paper is that the author uses the curvature function to produce observables that we can see in our own universe. These observables are the perceived distance to any object from a reference point and the measured angle observed between two observed objects with respect to the reference point. First, he uses this algorithm in a 1D universe to make the process easier to understand and visualize before it becomes more complicated when the 3D space is taken into account. He first generates many random points in flat space then applies the curvature function to simulate their positions in curved space. He shows that the distribution of distances from the reference point changes when the universe becomes curved instead of flat. He then repeats the method on a 3D universe. He first describes how the angle between two objects is affected by looking at each pair of adjacent data points. Next, he examines how the radial distance to the points is affected. Finally, he talks about the future implementations of CURVEFAM, and mentions that the main future update is that it will have a statistical fitting algorithm.

Recommendation

Accept with minor revision.

Recommendation Justification

Major points:

1. He says that “the effect of curvature on the distributions of distances is clear when the 1D universe is populated given an extreme curve in the form of a polynomial.” First of all, the word “extreme" is very ambiguous. There should be some kind of quantitative explanation to describe this curve. Along the same lines, what is the polynomial he uses to model the curvature? In order to be more assured that this is not low quality research, I would like to see evidence and explanation of the function used, especially since it is the entire basis of the project.
2. The biggest major point is that there is no discussion section. The author talks about the theory of what CURVEFAM does and how it is actually implemented, but I am not convinced that it is a “good” method. To be more specific, why is this method better than the other ones that already exist? I think that at the minimum there has to be some kind of comparison to results and models from other studies or there is not really a strong case for why I should put so much faith into this project as a legitimate tool for curvature modeling. There is one sentence at the end of the conclusion which says, “this is useful in the testing of theories regarding the effects of curvature on systems.” Honestly, this is probably one of the best sentences in the entire paper, and should have been the basis for a strong discussion section. I think that it is more important to learn how/why this method is so beneficial for science and what its applications are than how the method itself actually works.

Minor points:

1. The sentence in the introduction where the author attempts to define the top-down approach is very unclear. Because his entire paper is based on this approach, I think that it needs way more explanation and emphasis.
2. In Equation 1, he does not define what any of the variables mean. Even though it is not difficult to try to infer what they do mean, it is still important that they be defined in order to minimize any confusion.
3. In the introduction, he first introduces the word “observable,” but does not really define what it is. I think that it would clarify what an observable is categorized as, for example, is it a star, galaxy, supernova, etc?
4. In the 1-D model section, he said that he generates “a set of uniformly distributed points in flat space.” I think that this is extremely vague since there is no more explanation about it. There needs to be more detail about this, like any parameters used and the specific function in Python that was used for this. These details are essential so that I have a better understanding of what actually was done “under the hood.”
5. More proofreading was necessary, there were a few major grammatical errors.
6. Under the comprehensive model section, he first describes that the “reference point is now at the origin of a 3D universe uniformly populated with points.” This 3D universe definitely needs to be explained some more. I am coming in with the understanding that the universe is infinite and in that case there is no origin. It is important that he specifies the dimensions of this universe or say that it is finite, and then specify what the origin actually is.
7. There are a few issues with Equation 3. In the equation, he puts Y and T as a function of r and t. But then he says that r and t are the same thing. I might be confused here, but that needs to be clarified. He also says that r and t are interchangeable because of the finite speed of light, but I am quite confused on the connection here, and it also has to be explained in a little more detail.

<u>Evaluation</u>

1. Paper length
    a. This paper could use some more information about the application of the algorithm, to add maybe an extra page. Aside from this, I think it is very concise and does not waste paragraphs on things that do not need more explaining.
2. Title & abstract
    a. The title definitely reflects the summary and the abstract also properly reflects the goals and outcome of this project.
3. Contribution to science
    a. I think that if the project is extended and completed as described in the section about future implementations, then it will definitely be a significant contribution to science.
4. Level of English
    a. At some points, I have a little trouble understanding what the author is trying to say. After reading some of the paragraphs numerous times, I did have a better grasp, but there were certainly a few points that could have been explained better.

5. Literature
    a. In the introduction, there were many unsupported claims that clearly came from other sources. I know that they were true based on what we have learned in previous classes, but it is not like the author himself discovered some of the statements that he made, so more credit should have been given.
6. Presentation of results
    a. The results were represented well by most of the figures and were within reason of the methodology. However, it was clear that figures 5 and 6 were very important to the results, but the explanation for them could have been more clear and extensive. There should have also been a better explanation of the polynomial function.
7. Topic appropriateness
    a. The topic is appropriate for the journal.

Data Management

If made available, CURVEFAM would definitely be reusable as described in the Future Implementations section. Analysis can be redone with this algorithm and others can use it to verify his results.

Additional Comments

The author has done a very good job working with such a complicated topic in a very limited amount of time; it was a very impressive subject.