

The following report addresses the comments of reviewer 1:

I would like to thank the reviewer for the time spent on my paper and the comments on areas for improvement. The review greatly strengthens my work. In response to the reviewer's comments, I made the following changes:

- **All in-text citations should be parenthetical and in the proper AAS format: (Name year) rather than (Name (year)).**
  - The in-text citations have been fixed to follow AAS format.
- **The Kepler mission and Lightkurve should both be cited with papers when they are first mentioned.**
  - Citations have been added to the Kepler mission and Lightkurve.
- **More emphasis should be placed on the importance of this study as it contributes to science, especially since the values found are all extremely close to those found in previous literature. What does this study present that others have not, and what does it contribute?**
  - The study presents work that others have not because it uses new Kepler data to update the stellar parameters, and it conducts an in-depth asteroseismology analysis for the stellar parameters rather than a pipeline study. It also creates an Echelle diagram for the oscillation modes. This diagram has not yet been published in literature. I emphasized these contributions more throughout the paper.
- **The author uses “we” and “us” (plural) when there is only one author on the paper.**
  - Refute – the plural form recognizes those in the acknowledgements and the collaborative nature of science.
- **Table captions should be above the table, not below, and should be more descriptive rather than just giving the references cited.**
  - Table captions are now at the top of each tables.
- **“Their light curves exhibit two forms of variability...otherwise be unobservable” should be stated in the introduction and does not need to be mentioned in the abstract.**
  - I removed from the abstract and added to the introduction.

- **The author states “detailed asteroseismic analysis” but more details should be given on this analysis in the abstract.**
  - A brief explanation of the asteroseismic analysis now precedes the results presented in the abstract.
- **$v_{\max}$  and  $\Delta\nu$  should be defined and explained in the introduction.**
  - Refute – the intricacies of the asteroseismic analysis become more important in the data analysis section. I checked with numerous related studies, and they wait to define the parameters until the methodology section as well.
- **The time over which data were taken should be more specific. The exact dates should be given, rather than the year (“2009 to 2013”).**
  - Refute – the observations span 17 quarters, so the exact day and month of starting and stopping seem unimportant to the science of the study. I again checked with other similar studies, and they also omit the day/ month in the observation details.
- **What is winsorization? This should be explained.**
  - The edited light curve sets all normalized flux values above 1.0015 and below 0.9985 equal to one, the value to which all others are normalized. I previously mislabeled this process as winsorization. Winsorization would instead set all values above 1.0015 equal to 1.0015 and all values below 0.9985 equal to 0.9985. I decided against using winsorization because that would lead to a false excess of power at certain frequencies, which then impacts the determination of stellar parameters from the power spectrum.
- **The Lomb-Scargle method needs a citation.**
  - The Lomb-Scargle method now has a citation.
- **“The triangular method applies a boxcar filter to another boxcar.” This needs more explanation.**
  - The triangular smoothing method applies the box smoothing method twice with a specified filter width. I added a brief explanation of this method into my analysis section of the paper.
- **The definitions of the scaling relations and the definition of  $g^*$  should have equation numbers.**
  - The scaling relations and  $\log(g)$  definition now have equation numbers.

- **The scaling relations show up at the top of the next page, not after the paragraph where they are referenced.**
  - This formatting issue has been fixed.
- **The result of  $\log(g)$  should be given after the method of finding  $\log(g)$  is explained (at the end of the same paragraph).**
  - I rearranged the order of the  $\log(g)$  equation and the methods in response to this comment. They now flow more logically.
- **The uncertainty determination section can be combined with the Stellar Parameters section, and the uncertainty methods can be explained after the determination of  $v_{\max}$  and  $\Delta v$  is explained.**
  - The uncertainty determination methods now immediately follow the results for  $v_{\max}$  and  $\Delta v$ .
- **The process of how the Echelle diagram is made (e.g., did the author specify 5.47  $\mu\text{Hz}$  to make the diagram or did Lightkurve determine this?) and what it means should be explained a bit more.**
  - More explanation to the process of making an Echelle diagram and its meaning have been added to my paper. The value of 5.47 comes from the large separation frequency  $\Delta v$  determined in this study.
- **The percent differences should also be placed in new columns in Table 2 to facilitate comparisons.**
  - Table 2 now has these columns for the percent error values.
- **“A unique feature of heartbeat red giants is that they exhibit two forms of variability...” This sentence should be in the introduction, not the discussion.**
  - The sentence is now in the introduction.

The following report addresses the comments of reviewer 2:

I would like to thank the reviewer for the time spent on my paper and the comments on areas for improvement. The review greatly strengthens my work. In response to the reviewer's comments, I made the following changes:

- **The author should discuss more clearly what are the differences between her method and Beck et al.'s method because they both study the same object by using asteroseismology.**
  - Beck et al. uses an algorithm to estimate stellar parameters. I added a citation for this method from their study. I also elaborated on the differences between my study and previous studies.
- **The exact equations should be included. Also, the scaling relations are not a linear function of  $\nu_{max}$  and  $\Delta \nu$ , thus I think the error of the scaling relations should not be calculated the way the author point out. The author can refer to the “Simplification” section in this link [https://en.wikipedia.org/wiki/Propagation\\_of\\_uncertainty](https://en.wikipedia.org/wiki/Propagation_of_uncertainty) for the relationship between errors. If this is not what the author means, the author should explain it clearer.**
  - The method I described is the same one that you are referring me to. I added a clarification and fixed a typo to convey this better in my paper.
- **“... we conducted a detailed asteroseismic analysis of KIC 7431665 ...”: the author should add the term “heartbeat binary” before “KIC 7431665” to emphasize the type of the object.**
  - This is now changed.
- **In the abstract, the author should state by words what all mathematical notations are (for example, Radius R, mass M, etc.)**
  - The notations are now defined by words in the abstract.
- **“We hope future studies will conduct a ... oscillation mode.”: I think this should not be in the abstract. The abstract should focus on the analysis and the result of the work, rather than suggestion/ expectation for further studies.**
  - The sentence was removed from the abstract.

- **“... from the discovery of red giant heartbeat stars.”: need citation for the discovery.**
  - Yes, a citation was added.
- **“Convection in the envelope generates solar-like oscillations via the ...”:** the author should briefly explain what a solar-like oscillation is or what is its mechanism.
  - Solar-like oscillations are driven by convection in the envelope. I made a change to clarify this.
- **“In the case of red giants ... largest amplitudes in the core.”: need citation.**
  - Yes, a citation was added.
- **“... derive from various scaling relations.”: the author should briefly explain what scaling relations mean.**
  - Refute – scaling relations will be understood by anyone in the field. I also reference the equations to specify what I mean by them.
- **“The scaling relations for stellar mass ... from the power spectrum.”: need citation.**
  - Yes, a citation was added.
- **“The effective temperature applied in this study is 4580K ...” : the author should explain why and how the effective temperature is applied to this study (i.e what do we need it for?).**
  - The effective temperature is needed for the scaling relations. I added a clarification of this into the paper.
- **“We combine the individual light curve ... all observation windows”:** the author should refer to her **Figure 1** after this sentence.
  - I added a reference to Figure 1 here.
- **“We remove these eclipses by winsorization ...”:** the term “winsorization” should be explained clearer. Also, I think the author should include two light curves before and after the winsorization reduction so we can see what the winsorization’s effect on the light curve is.
  - The edited light curve sets all normalized flux values above 1.0015 and below 0.9985 equal to one, the value to which all others are normalized. I previously mislabeled this process as winsorization. Winsorization would instead set all values above 1.0015 equal to 1.0015 and all values below 0.9985 equal to 0.9985. I decided against using winsorization because that would lead to a false excess of

power at certain frequencies, which then impacts the determination of stellar parameters from the power spectrum.

- **“*The LIGHTKURVE packages...*”**: need citation for the package.
  - Yes, a citation was added.
- **“*... the Lomb-Scargle method...*”**: need citation for the method.
  - Yes, a citation was added.
- **“*We remove this noise by flattening the power spectrum with a triangular filter...*”**: the author should show a flattened power spectrum in the paper.
  - The power spectrum included in the paper is flattened.
- **“*... for KIC 7431665 with the following scaling relations: ...*”**: the equations should be listed right after the colon (instead of on the next page) – format problem.
  - The formatting problem is now fixed.
- **“*... and the generate a random sample from that data.*”**: the author should explain clearer what she means “generating a random sample from that data”. For example, what distribution that random sample follows, or what parameters of the sample are taken from the data.
  - Refute – this is a general bootstrapping technique. I added a bootstrapping citation to my paper for any further clarifications.
- **A brief explanation of the purpose of Échelle diagram should be given. Also, the author should explain what each axis in the Échelle diagram represents. what is the oscillation mode l? The author should explain it along with the explanation of oscillation modes p and g in the introduction.**
  - More explanation to the process of making an Echelle diagram and its meaning have been added to my paper.
- **“*For the comparison to Beck et al (2014) ... For the comparison to Manuel & Hambleton (2018) ...*”**: the author should briefly mention the methods of Beck et al (2014) and Manuel & Hambleton (2018) to understand why the results are slightly different.
  - A brief description of the methods for each study was added. The typo citing Manuel & Hambleton (2018) was fixed to correctly cite Yu et al. (2018).

- ***“An interesting binary analysis would ...”*: I don’t think “interesting” is a good word to use in this sentence.**
  - I removed the word “interesting” from this sentence.
- ***“The uncertainty ranges result from the standard error and error propagation.”*: the author should also briefly remind the readers that bootstrapping resampling process is the method to calculate the standard error.**
  - The section now has a brief reminder of the bootstrapping resampling process.

The following report addresses the comments of reviewer 3:

I would like to thank the reviewer for the time spent on my paper and the comments on areas for improvement. The review greatly strengthens my work. In response to the reviewer's comments, I made the following changes:

- **Throughout the text, there are instances where in text citations are not placed in parenthesis when they should be**
  - The parenthetical citations now follow the AAS guidelines with parentheses enclosing the author's name and year.
- **There are many technical details in the introduction that are mentioned with no explanation. Why do p modes dominate solar like oscillations in the convective envelope while g modes have large amplitudes in the core for red giants? How does temperature determine the oscillation mechanism of a star? There are many statements in the introduction that should be explained further. This happens again in Section 3 with certain details or features that are mentioned but not explained. For example, what is winsorization and the region of excess power?**
  - I address these details in the edits of my paper. The only one I did not include was winsorization because of the misuse of that term. The edited light curve sets all normalized flux values above 1.0015 and below 0.9985 equal to one, the value to which all others are normalized. I previously mislabeled this process as winsorization. Winzorization would instead set all values above 1.0015 equal to 1.0015 and all values below 0.9985 equal to 0.9985. I decided against using winsorization because that would lead to a false excess of power at certain frequencies, which then impacts the determination of stellar parameters from the power spectrum.
- **Equation 1 is not placed in the correct location. There seems to be a break in section 3 where the equation would go but then the equation is displayed on the next page. Also, the solar symbol,  $\odot$ , should be placed in the subscript, not the superscript.**
  - The formatting issue with Equation 1 is now fixed, but I left the solar symbol as a superscript when I already had a subscript.



- **The Echelle diagram needs to be better explained. It is hard to understand what is actually being plotted and what this plot demonstrates.**
  - o More explanation to the process of making an Echelle diagram and its meaning have been added to my paper.
- **In the Discussion, there is a comparison to the results of Manuel & Hambleton (2018). Did they also study KIC 7431665, are there values for KIC 7431665?**
  - o The typo citing Manuel & Hambleton (2018) was fixed to correctly cite Yu et al. (2018).
- **There needs to be a larger, in-depth discussion of Heartbeat red giant stars. There is very little stated on these stars even though the project is an asteroseismic analysis of heartbeat binaries. There needs to be more stated about the theory of these objects and a better literature review on the physics of Heartbeat stars.**
- **There are many citations missing. There needs to be a citation for Kepler and KIC and also all the programs used, including Lighkurve, Lomb-Scargle, bootstrapping etc.**
  - o Yes, I added in these necessary citations.
- **There is not much to distinguish this paper from the previous studies conducted on KIC 7431665. There seems to have already been two in depth asteroseismic studies on the target that already used Kepler data. This study seems to have just repeated their analyses. The results seem to confirm this as all the determined values are very similar to the values in the literature. There needs to be something else added to separate this study from what has already been done. Perhaps a discussion/analysis of internal structure or the oscillation modes. I also wonder if there are any other sources of new observations (perhaps TESS). It seems like asteroseismology has already been conducted on this star with the Kepler observations, a study with new measurements would be interesting to reconfirm the literature parameters or analyze any changes. As of now, it is unclear why another asteroseismic study on the Kepler data is needed.**
  - o The study presents work that others have not because it uses new Kepler data to update the stellar parameters, and it conducts an in-depth asteroseismology analysis for the stellar parameters rather than a pipeline study. It also creates an

Echelle diagram for the oscillation modes. This diagram has not yet been published in literature. I emphasized these contributions more throughout the paper.

- *TESS* data is currently insufficient for an asteroseismic analysis because only one observation set is available. I constructed a power spectrum from *TESS* data and found no useful parameters for an analysis to be completed. Additionally, the methods used to determine stellar parameters from the *Lightkurve* package do not work well for the low signal to noise ratios of *TESS*.

- **In the beginning of section 4, one of the results of this project is that it “demonstrates the necessity for single-star studies in refining the results of pipeline projects ...”. I feel that this result is not met by this study. As stated before, this study conducted asteroseismology using the same data as other studies, and therefore there were minimal changes in the parameters determined. Since the determined parameters are so close to the previous values, this claim does not hold.**

- I agree. I removed this sentence and placed my emphasis in the discussion elsewhere.

The following report addresses the comments of reviewer 4:

I would like to thank the reviewer for the time spent on my paper and the comments on areas for improvement. The review shared helpful advice, all of which I responded to before resubmission.

The following changes have been made:

- **Introduction: “...red giant dominates the observational properties of a heartbeat binary unless the companion star is of high density, such as a white dwarf.” It may be helpful to characterize or just mention the companion star in this binary system. I was left wondering what type of star the companion was and if/how it affected the system as a whole. Do we not know much about it? Can we infer anything about it by studying the red giant?**
  - Literature does not yet have any information about the companion star. A future study conducting a binary analysis would resolve this issue.
- **A question of little concern, but is there a reason why the author did not consider more recent *TESS* data over *Kepler* data?**
  - *TESS* data is currently insufficient for an asteroseismic analysis because only one observation set is available. I constructed a power spectrum from *TESS* data and found no useful parameters for an analysis to be completed. Additionally, the methods used to determine stellar parameters from the *Lightkurve* package do not work well for the low signal to noise ratios of *TESS*.
- **“BKJD” is never defined. Assuming it’s a certain type of Julian Date?**
  - BKJD stands for Barycentric Kepler Julian Date. I added this to my paper for clarification. Essentially, it takes the Barycentric Julian Date (BJD) and subtracts 2454833.0, which shifts the zero point.
- **“WinsORIZATION” is also not a commonly used term and may be beneficial to define.**
  - The edited light curve sets all normalized flux values above 1.0015 and below 0.9985 equal to one, the value to which all others are normalized. I previously mislabeled this process as winsORIZATION. WinsORIZATION would instead set all values above 1.0015 equal to 1.0015 and all values below 0.9985 equal to 0.9985. I decided against using winsORIZATION because that would lead to a false excess of

power at certain frequencies, which then impacts the determination of stellar parameters from the power spectrum.

- **“...shifts windows of 25  $\mu\text{Hz}$  along the data to evaluate the correlation of the data with itself.” Is there a reason the shift is by 25  $\mu\text{Hz}$ ? Is this just a small enough frequency shift to find the correlation?**
  - The filter width depends on the star being analyzed. *Lightkurve* defaults to 25 microhertz for Red Giants and 250 microhertz for Main Sequence stars.
- **“The diagram indicates oscillation modes of  $l=1, 2,$  and  $0$  corresponding to the ridges from left to right.” I wish the author would have described what “ridges” mean in reference to this diagram. Are they at the points when the SNR is highest? In addition, what is the significance of these oscillation modes? What do the numbers 1, 2, and 0 mean? Why is the author confident of performing actions on  $\Delta \nu$  from the observation of these ridges?**
  - More explanation to the process of making an Echelle diagram and its meaning have been added to my paper.

The following report addresses the comments of reviewer 5:

I would like to thank the reviewer for the time spent on my paper and the comments on areas for improvement. The review shared helpful advice, all of which I responded to before resubmission.

The following changes have been made:

- **Tables could use descriptions**
  - The tables include descriptions in this resubmission. I placed a description in a caption at the top of each table.
- **A brief description of the LIGHTKURVE package might aid in understanding. Is it standalone software or part of some other program?**
  - *Lightkurve* is a python package. I added the citation for *Lightkurve* to my paper for any further clarifications.
- **Plot size can be increased for better clarity and detail.**
  - The plot size of each figure was increased. The details of the plots now show up more clearly.
- **Unclear how the Échelle diagram indicates modes of  $l=1,2$ , and 0**
  - A more detailed explanation of the Échelle diagram has been added to the paper. This includes the methods behind recognizing the oscillation modes.
- **Paper would benefit from an additional proofread**
  - I proofread the paper numerous times before submitting this most recent version. Any missed typos/ grammatical errors will (hopefully) be caught for the next draft or by the editor if this version is accepted to the journal.

The following report addresses the comments of reviewer 8:

I would like to thank the reviewer for the time spent on my paper and the comments on areas for improvement. The review provided particular help in identifying parts of the paper in need of more explanation. The following changes have been made:

- **The author mentions multiple times an “asteroseismic analysis.” What does this entail? Is it a general method of analysis with specific steps to follow or does it differ for each scientist?**
  - The steps in the process of an asteroseismic analysis vary from one study to the next, but all asteroseismic analyses take advantage of stellar oscillations to gain information about a star, such as its internal structure or fundamental parameters). In the abstract, I added a sentence briefly overviewing the particular asteroseismic analysis methods for this study. The analysis portion of the paper expands more on this methodology.
- **What are scaling relations?**
  - In general, scaling relations are equations that show how one variable depends on others. In the field of asteroseismology, these are used to determine stellar parameters. I included equations for the scaling relations used in this study.
- **What are BKJD days?**
  - BKJD stands for Barycentric Kepler Julian Date. I added this to my paper for clarification. Essentially, it takes the Barycentric Julian Date (BJD) and subtracts 2454833.0, which shifts the zero point.
- **The author states that “the triangular method applies a boxcar filter to another boxcar.” What is the triangular method? What is a boxcar filter and what does she mean she is applying it to another boxcar?**
  - The triangular smoothing method applies the box smoothing method twice with a specified filter width. I added a brief explanation of this method into my analysis section of the paper.
- **I do not really understand the entire explanation of autocorrelation. What it is and the point of it I think should be explained more clearly.**

- The point of the autocorrelation function in this study is to extract the values for  $\Delta\nu$  and  $\nu_{\max}$  from the power spectrum. I added more details on the methods of the autocorrelation function to the paper to provide a better explanation of the methods. The *Lightkurve* package carried out this method.
- **What is  $\Delta\nu$ ? I do not think it was defined or explained until the very end of the paper.**
  - The value for  $\Delta\nu$  refers to the large frequency separation between consecutive radial modes. I added a definition of  $\Delta\nu$  to the abstract and the methodology sections.
- **Regarding the Echelle diagram, I am not sure if this is because I am not an expert in this field, but I am not completely clear on what it is based on the explanation the author provides. More specifically, what is the point of it and what is it supposed to be showing that is relevant to the project?**
  - More explanation to the process of making an Echelle diagram and its meaning have been added to my paper.
- **In the abstract, the author lists the results of the study; however, I am not sure about what some of the variables / units mean.**
  - I added each variable name in words to my abstract, but I left the units the same. The units follow astronomy convention.
- **There are some instances of incorrect grammar and odd wording, so the author should proofread again.**
  - The paper has been proofread and edited again.
- **In Table 1 and Table 2, the only thing in the caption is the references. I would like to see more information about what the tables themselves are showing rather than just the references.**
  - The tables include descriptions in this resubmission. I placed a description in a caption at the top of each table.
- **In the introduction, there is very little background information about the star system itself that is being studied, aside from a few parameters pulled from another paper. It would be beneficial to the reader if the author described the system a little bit more, so I have more context about what is being done.**

- I added the type of star, right ascension, and declination values for the star in the introduction.
- **I think there should be more information about the data reduction and processing done on this data, if it is available somewhere.**
  - Refute – I included references for the *Kepler* mission for any further inquiries about the data collection.
- **The software used should be specified. I know that it is Python, but it is still important to state in case someone does not.**
  - I added a specification of python when I first introduced the *Lightkurve* package.
- **All the equations could be formatted a little bit better and actually labeled as equations.**
  - All equations now have numbers to label them as equations.
- **Two big studies are mentioned that the author compares her results to. However, I do not know what these studies did. In order to trust that the results can actually be compared, there should be more information about the studies, so I know more about the data used, methods, etc.**
  - The studies conducted a largescale asteroseismic analyses of numerous stars at one time. The study by Beck et al. is detailed in the introduction, and I added a brief description of the study by Yu et al. to my discussion before the comparison.