

FOURIER TRANSFORMS AND SIGNAL PROCESSING

Due date: 04/21/2022

Fourier transforms are one of the most commonly used signal processing tools. In a nutshell, think of a time-domain function $h(t)$ and a corresponding frequency-domain function $H(f)$ as two representations of a single function, with the following mapping that lets you go back and forth between the two:

$$H(f) = \int_{-\infty}^{\infty} h(t)e^{i2\pi ft} dt, \quad h(t) = \int_{-\infty}^{\infty} H(f)e^{-i2\pi ft} df.$$

For N discrete data points $h(t_k) \equiv h_k$, the discrete Fourier transform is given by:

$$H(f_n) \equiv H_n = \sum_{k=0}^{N-1} h_k e^{i2\pi kn/N},$$

and its corresponding inverse is given by:

$$h(t_k) \equiv h_k = \frac{1}{N} \sum_{n=0}^{N-1} H_n e^{-i2\pi kn/N}.$$

Of course, as we discussed in class, no one in their right mind would ever do DFT instead of FFT if they can help it.

- a) Compute and plot the Fourier transform and the inverse Fourier transform for the following functions: sine, cosine, square pulse, triangle, delta. Compute the (one-sided) power spectral density for all functions. This should give you a clear idea of the Fourier transform.
- b) Cook up a mixture of sines and cosines, i.e. $h(t) = \sin \pi t + 3 \sin 3\pi t + 5 \cos 5\pi t$. Try the same with non-integer frequencies. Demonstrate aliasing and verify that spectral power is conserved.
- c) Analyze the sound of a guitar (the wav and the text files are on the course homepage).
- d) Listen to Bach's Partita in the uncompressed mp3 format. Then listen to a 2.3s excerpt that has been sampled at different rates: 882, 1378, 2756, 5512, 11025 and 44100. Once you hear what the effect of under-sampling is, quantify it by the Fourier transform. All data (in both mp3 and ascii) are available on the course webpage.
- e) Compute the autocorrelation function of the sound of boiling water. If you are so inclined, acquire your own data, otherwise you can use `boiling.data` from the course homepage. *Hint*: the autocorrelation function will likely drop rapidly and you should use the log scale to visualize it.
- f) Using convolution, predict the shape of a spectral line out of a diffraction-limited spectrograph. *Hint*: show (or assume) that the response function is a gaussian.