DISCRETE FOURIER TRANSFORMS

Due date: 04/05/2016

Fourier transforms are one of the most commonly used signal processing tools. In a nutshell, think of a time-domain function $h(t)$ and a corresponding frequency-domain function $H(f)$ as two representations of a single function, with the following mapping that lets you go back and forth between the two:

$$H(f) = \int_{-\infty}^{\infty} h(t) e^{i2\pi ft} dt, \quad h(t) = \int_{-\infty}^{\infty} H(f) e^{-i2\pi ft} df.$$

For N discrete data points $h(t_k) \equiv h_k$, the discrete Fourier transform is given by:

$$H(f_n) \equiv H_n = \sum_{k=0}^{N-1} h_k e^{i2\pi kn/N},$$

and its corresponding inverse is given by:

$$h(t_k) \equiv h_k = \frac{1}{N} \sum_{n=0}^{N-1} H_n e^{-i2\pi kn/N}.$$

a) Compute and plot the Fourier transform and the inverse Fourier transform for the following functions: sine, cosine, square pulse, triangle, delta. Compute the (one-sided) power spectral density for all functions. This should give you an idea of what the Fourier transform does.

b) Cook up a mixture of sines and cosines, i.e. $h(t) = \sin \pi t + 3 \sin 3\pi t + 5 \cos 5\pi t$. Try the same with non-integer frequencies. Can you detect aliasing?

c) Analyze the sound of a guitar (the wav and the text files are on the course homepage). Can you detect harmonics?

d) Listen to Bach's Partita in the uncompressed mp3 format. Then listen to a 2.3 s exerpt that has been sampled at different rates: 882, 1378, 2756, 5512, 11025 and 44100. Once you hear what the effect of undersampling is, quantify it by the Fourier transform. All data (in both mp3 and ascii) are available on the course webpage.