REAL PENDULUM

Due date: 3/15/2016

Now that you know about dynamical chaos, it is time to explore a real mathematical pendulum, where by *mathematical* we mean a bob with mass m suspended on a massless string of length l. As you have seen in class, you can approach this problem either by considering all the forces explicitly, or by invoking the Hamilton principle. For each of the assignments below be sure to explore the phase diagrams, the existence of any closed orbits, chaotic behavior. Animate each problem to make sure that the results are sane. A word of caution: these animations are just super-cool: you can spend hours just staring at them. Don't forget that you need to explore these systems dynamically, not visually. ;)

1. Start with an ideal mathematical pendulum ($l = \text{const.}$), but this time do not do the $\sin\theta \approx \theta$ approximation. How does the oscillation period depend on the amplitude?
2. Now allow for the string to stretch and shrink. Use Hooke's law to describe its motion. Note that a real string would not really behave like a spring, but assume for a moment that it does. Then figure out how to make this more realistic. *Hint: does the spring constant k need to be really constant?*
3. Next up: turn off stretching of the string altogether, and replace it with freefall.
4. Finally, add attenuation (air resistance) into the system that is linearly proportional to the velocity.
5. Once you have all the building blocks of a real pendulum, you can easily create complex structures, such as coupled oscillators, vertically stacked pendula, Newton's cradle, etc. Let your imagination go crazy.
6. *Extra credit:* determine the Lyapunov exponent for chaotic orbits of an undampened oscillator that is allowed to bounce.