THE KEPLER PROBLEM

Due date: 2/2/2016

The two-body problem in a central potential can be described analytically. The orbital equation cannot be written explicitly as function of time (dr/dt), but it can be written explicitly as function of angle (dr/dv). However, in practice we often want to compute the location of a moving object as function of time. For such cases we have two possible approaches: solve the problem iteratively with anomalies or solve the differential equation numerically. We will apply both methods to the motion of Halley's comet and stability of the Solar system.

1. **Iterative solution.** Halley's comet might be the most famous comet of all time. Its perihelion distance is 0.587 au, eccentricity is 0.967 and the orbital period is 76 years. Using Newton-Raphson's method, compute the distance of Halley's comet from the Sun as function of time. Plot the orbit.

2. **Differential solution.** Using a 4-th order Runge-Kutta method, compute the orbit of Halley's comet and compare it with the orbit obtained by iterative solution. Plot the accumulated error as function of time (or the number of orbits).

3. **Stability.** Using two or three different integrators (i.e. `euler`, `rk45`, `rk8pd`, `odeint`, ...) compute the time (or the number of orbits) it takes for the Solar system to collapse.

A quick recap: the equation of an elliptical orbit is:

$$r(v) = \frac{a(1-e^2)}{1 + e\cos v}, \tag{1}$$

where a is the semi-major axis, e is orbital eccentricity and v is true anomaly. The true anomaly is measured from perihelion and can be related to time via the eccentric and mean anomalies:

$$\tan\frac{v}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}, \qquad M = E - e\sin E = \frac{2\pi(t-\tau)}{P}, \tag{2}$$

where E is the eccentric anomaly, M is the mean anomaly, τ is the reference time at perihelion and P is the orbital period. Equation $M = E - e\sin E$ cannot be solved analytically for E, thus it needs to be solved iteratively, using the Newton-Raphson method. This is also the reason why $r(t)$ cannot be written out explicitly.